Identification of Structural Non-linearities Using Describing Functions and Sherman-Morrison Method

Mehmet Bulent Özer[a], H. Nevzat Özgüven[b], Thomas J. Royston[c]

a) Post-Doctorate Researcher, University of Illinois at Chicago
The Dept. of Mech. and Indust. Eng. (M/C 251), 2039 ERF, 842 W. Taylor Street
Chicago, Illinois 60607

b) Professor, Mech. Eng. Dept., Middle East Technical University, 06531 Ankara, Turkey

c) Professor, University of Illinois at Chicago
The Dept. of Mech. and Indust. Eng. (M/C 251), 2039 ERF, 842 W. Taylor Street
Chicago, Illinois 60607

ABSTRACT

In this study, a new method for type and parametric identification of a non-linear element in an otherwise linear structure is introduced. This work is an extension of a previous study in which a method was developed to localize non-linearity in multi degree of freedom systems and to identify type and parameters of the non-linear element when it is located at a ground connection of the system. The method uses a describing function approach for representing the non-linearity in the structure. The Sherman-Morrison matrix inversion method is used in the present study to put the response expression in a form where the non-linearity term can be isolated. Using measured responses one can calculate the value of the describing function representation of the non-linear element and hence perform the identification. This new method can be used for type and parametric identification of a non-linear element between any two coordinates of the system. Case studies are given to demonstrate the applicability of the method.

1. INTRODUCTION

Engineering structures generally exhibit some degree of non-linearity. Localization and identification of non-linearity by using experimentally measured response is an important issue for several reasons. For instance, in modal updating, a non-linearity in the system will make any linear model incorrect and model updating would not correct the problem, but would require different corrections at different forcing levels.

There has been extensive research on localization and identification of non-linearity in structures. Studies in this field can be classified in two groups: time domain and frequency domain techniques. Time domain techniques generally focus on time series [1] and force state mapping [2], [3] and [4]. Some of the frequency domain techniques in which first order or higher order harmonic responses are used to localize and/or identify non-linearity in structures include the work of Tanrıkulu and Özgüven [5], Vakakis and Ewins [6], Lin and Ewins [7], Şanlıtürk and Ewins [8], Richards and Singh [9], and Özer and Özgüven [10].

In this work the method developed in an earlier study [10] will be used to localize non-linearity in multi degree of freedom systems by using measured harmonic responses. The method will then be extended to identify the type and the parametric values of the non-linear element at a structural system where there is a single non-linear element between any two coordinates of the system. The identification method presented in the earlier study [10] is restricted to systems with non-linear elements located at ground connections only. Some numerical case studies will be presented to demonstrate the application of the method.

2. THEORY

The method developed is based on expressing the non-linear forcing vector in a matrix multiplication form in the differential equation of motion of a multi degree of freedom (MDOF) non-linear system with harmonic excitation. This has been first achieved by Budak and Özgüven [11, 12] in their studies analyzing harmonic vibrations of non-linear MDOF systems, and then the method is generalized for any type of non-linearity by Tanrıkulu, et. al [13] by using describing functions. This representation has been used in several other studies on harmonic vibrations of non-linear MDOF systems. Some of the work in this field can be listed as follows: Tanrıkulu and Özgüven[14]; Cömert and Özgüven [15]; Kuran and Özgüven [16]; Ferreira and Ewins [17], Özer [18]; Platten, et al. [19].

2.1. Non-linearity Matrix

Consider the differential equation of motion of a non-linear MDOF system with harmonic excitation

$$[\mathbf{M}]\cdot\{\ddot{\mathbf{x}}\}+[\mathbf{C}]\cdot\{\dot{\mathbf{x}}\}+[\mathbf{K}]\cdot\{\mathbf{x}\}+\{\mathbf{N}\}=\{\mathbf{f}\} \tag{1}$$

where [$\mathbf{M}$], [$\mathbf{C}$], and [$\mathbf{K}$] are mass, viscous damping and stiffness matrices of the linear system, respectively. {$\mathbf{f}$} and {$\mathbf{N}$} represent the external forcing and non-linear internal forcing and i represents the unit imaginary number. For a harmonic excitation in the form of

$$\{\mathbf{f}\}=\{\mathbf{F}\}\cdot e^{i\cdot\omega\cdot t} \tag{2}$$

where {$\mathbf{F}$} denotes the amplitude vector of forcing, the response can be assumed to be

$$\{\mathbf{x}\}=\{\mathbf{X}\}\cdot e^{i\cdot\omega\cdot t} \tag{3}$$

where {$\mathbf{X}$} is the vector of response amplitudes. Since the response is assumed to be harmonic, the internal non-linear forces can be assumed to be harmonic, which can be written as:

$$\{\mathbf{N}\}=\{\mathbf{G}\}\cdot e^{i\cdot\omega\cdot t} \tag{4}$$

Substituting equations (2), (3) and (4) in equation (1) yields

$$\left(-\omega^2\cdot[\mathbf{M}]+i\cdot\omega\cdot[\mathbf{C}]+[\mathbf{K}]\right)\cdot\{\mathbf{X}\}+\{\mathbf{G}\}=\{\mathbf{F}\} \tag{5}$$

The amplitude of internal forcing vector can be expressed [10-12] as

$$\{\mathbf{G}\}=[\boldsymbol{\Delta}]\cdot\{\mathbf{X}\} \tag{6}$$

where [Δ] is response dependent, the so called 'non-linearity matrix'. The 'non-linearity matrix' can be formed by using describing function representations of internal non-linear forces. For details of how describing functions are employed in the calculation of 'non-linearity matrix' one should refer to work of Tanrıkulu and Özgüven [14] and Tanrıkulu et. al [13]. This representation makes it possible to make the harmonic vibration analysis of non-linear MDOF systems with any type of non-linearity provided that convergence is obtained in the iterative solution. This is basically the application of the harmonic balance method and describing functions to non-linear MDOF systems. Moreover, by using this representation it is also possible to apply almost all the techniques developed for linear multi degree of freedom systems to non-linear multi degree of freedom systems, provided that non-linearity is not so high that harmonic response assumption holds true.

From equations (5) and (6) the relation between harmonic forcing and response can be written as:

$$\left(-\omega^2 \cdot [\mathbf{M}] + i \cdot \omega \cdot [\mathbf{C}] + [\mathbf{K}] + [\boldsymbol{\Delta}]\right)\{\mathbf{X}\} = \{\mathbf{F}\} \tag{7}$$

Then $\{\mathbf{X}\}$ can be written as

$$\{\mathbf{X}\} = [\boldsymbol{\theta}] \cdot \{\mathbf{F}\} \tag{8}$$

where $[\theta]$ is the pseudo receptance matrix of the non-linear structure, given by

$$[\boldsymbol{\theta}] = \left(-\omega^2 \cdot [\mathbf{M}] + \mathrm{i} \cdot \omega \cdot [\mathbf{C}] + [\mathbf{K}] + [\boldsymbol{\Delta}]\right)^{-1} \tag{9}$$

$[\theta]$ is called 'pseudo' receptance since it is the receptance matrix of the system only for a specific response level. The receptance matrix $[\boldsymbol{\alpha}]$ of the linear part of the same system can be written as

$$[\boldsymbol{\alpha}] = \left(-\omega^2 \cdot [\mathbf{M}] + \mathbf{i} \cdot \omega \cdot [\mathbf{C}] + [\mathbf{K}]\right)^{-1} \tag{10}$$

From equations (9) and (10), $[\boldsymbol{\Delta}]$ can be obtained as:

$$[\boldsymbol{\Delta}] = [\boldsymbol{\theta}]^{-1} - [\boldsymbol{\alpha}]^{-1} \tag{11}$$

Post multiplying both sides of equation (11) by $[\theta]$ yields

$$[\boldsymbol{\Delta}] \cdot [\boldsymbol{\theta}] = [\mathbf{I}] - [\mathbf{Z}] \cdot [\boldsymbol{\theta}] \tag{12}$$

where $[\mathbf{Z}]$ is the dynamic stiffness matrix of the linear part of the system, and is given by

$$[\mathbf{Z}] = [\boldsymbol{\alpha}]^{-1} = \left([\mathbf{K}] - \omega^2 \cdot [\mathbf{M}] + \mathrm{i} \cdot \omega \cdot [\mathbf{C}]\right) \tag{13}$$

Equation (12) is the starting point for determining harmonic response of a non-linear MDOF system by using the receptances of the corresponding linear system.

2.2. Evaluation of Non-linearity Matrix Using Single Harmonic Response

Tanrikulu et. al. [13] proposed the use of describing functions for the evaluation of the $[\Delta]$. The elements of $[\Delta]$ can be found from υ_{rj} as follows:

$$\Delta_{rr} = \upsilon_{rr} + \sum_{\substack{j=1 \\ j \neq r}}^{n} \upsilon_{rj} , \tag{14}$$

$$\Delta_{rj} = -\upsilon_{rj} , \tag{15}$$

where υ_{rj} is the harmonic describing function representation of the non-linear force and can be evaluated using the expression below:

$$\upsilon_{rj} = \frac{\mathrm{i}}{\pi Y_{rj}} \int_0^{2\pi} N_{rj} e^{\mathrm{i}\psi} d\psi , \tag{16}$$

where N_{rj} is the non-linear forcing between the r^{th} and j^{th} coordinates and Y_{rj} is the amplitude of the inter-coordinate displacement

$$Y_{rj} = \begin{Bmatrix} \text{if } r \neq j & X_r - X_j \\ \text{if } r = j & X_r \end{Bmatrix} \tag{17}$$

and

$$\psi = \omega t \; . \tag{18}$$

It should be noted that the harmonic describing functions of many systems have already been derived; a table of describing functions for different types of non-linearities is given by Gelb and van der Velde [5]. Common types of structural non-linearities are given in Table 1.

If the type of non-linearity is known, using the describing functions one can find the describing function representation of the non-linearity and form the Δ matrix using equations (14-18). It should be noted that the expressions given above for the elements of [Δ**]** are also a function of the response itself. Therefore in order to calculate the response one should employ an iterative procedure. The initial guess can start with the response of the linear system. First, the initial guess is used in the Δ expression (Equations 14-18) and then one can solve for the non-linear response (Eq. 9). In the next iteration step, the calculated responses are used in obtaining the elements of [Δ**]** and non-linear responses are recalculated. This procedure goes on until a sufficient convergence criterion is satisfied for the non-linear responses. It should be noted that the response is a single harmonic. However, it is always possible to include the higher harmonics as explained, for instance in [16].

3. IDENTIFICATION OF STRUCTURAL NON-LINEARITIES USING DESCRIBING FUNCTIONS AND SHERMAN-MORRISON METHOD

3.1. Localization of Structural Non-linearity

Özer and Özgüven [10] introduced non-linearity numbers that can be obtained from harmonic vibration measurements, in order to localize any non-linear element in the system:

Let us consider the ith column of equation (12), which will give

$$[\boldsymbol{\Delta}]\cdot\{\boldsymbol{\theta}^i\} = \{\mathbf{e}^i\} - [\mathbf{Z}]\cdot\{\boldsymbol{\theta}^i\} \tag{19}$$

where $\{e^i\}$ is a vector of which ith element is unity while all other elements are zero, and $\{\theta^i\}$ is the ith column of $[\theta]$. The rth row of equation (19) gives,

$$[\boldsymbol{\Delta}_r]\cdot\{\boldsymbol{\theta}^i\} = \delta_{ir} - [\mathbf{Z}_r]\cdot\{\boldsymbol{\theta}^i\} \tag{20}$$

where $[\boldsymbol{\Delta}_r]$ and $[\mathbf{Z}_r]$ represent the rth rows of $[\boldsymbol{\Delta}]$ and $[\mathbf{Z}]$, respectively, and δ_{ir} is the Kronecker Delta function. The term $\delta_{ir} - [\mathbf{Z}_r]\cdot\{\boldsymbol{\theta}^i\}$, is called "non-linearity number (NLN)". If we consider the rth coordinate of the system, NLN_r is an indication of non-linear element(s) connected to the rth coordinate. This term is zero if the rth row and column of [**Δ**] have all zeros. Any nonzero element in the rth row or column of [**Δ**] will yield a nonzero non-linearity number for the rth coordinate, since NLN_r can be written, from equation (20), as

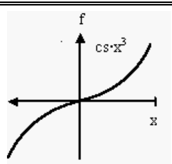
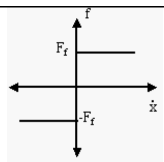
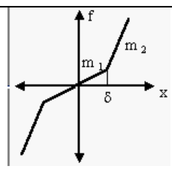
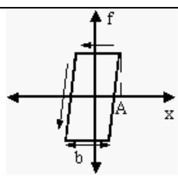
$$NLN_r = \Delta_{r1}\cdot\theta_{1i} + \Delta_{r2}\cdot\theta_{2i} + ... + \Delta_{rn}\cdot\theta_{ni} \tag{21}$$

Here i can be any coordinate. However, in calculating NLN_r from experimental measurements, the right hand side of equation (20) will be used:

$$NLN_r = \delta_{ir} - Z_{r1}\cdot\theta_{1i} - Z_{r2}\cdot\theta_{2i} - .... - Z_{rn}\cdot\theta_{ni} \tag{22}$$

Then, NLN_r can be calculated from harmonic vibrations measured at all coordinates connected to the r^{th} coordinate, while the system is excited at coordinate i. The measurements are carried out at high and low forcing levels. $[\mathbf{Z}_r]$ is obtained from low forcing measurements while $\{\theta^i\}$ is obtained from high forcing response.

Table 1. Describing Function Representations of Common Structural Non-linear Elements

TYPE OF NONLINEARITY		**DESCRIBING FUNCTION REPRESENTATION**
	Cubic Stiffness	$\text{Desc} = \frac{3}{4}A^2$
	Coulomb Damping	$\text{Desc} = j\frac{4F_f}{\pi A}$
	Piecewise Stiffness	$\text{Desc} = (m_1 - m_2)\frac{2}{\pi}\left(\arcsin\left(\frac{\delta}{A}\right) + \frac{\delta}{A}\sqrt{1-\left(\frac{\delta}{A}\right)^2}\right) + m_2$
	Friction Controlled Backlash	$\text{Desc} = \frac{1}{2}\left(1+\frac{2}{\pi}\left(\arcsin\left(1-\frac{b}{A}\right) - \left(1-\frac{b}{A}\right)\sqrt{1-\left(1-\frac{b}{A}\right)^2}\right)\right) - j\frac{1}{\pi}\left(\frac{2b}{A} - \left(\frac{b}{A}\right)^2\right)$

3.2. Analysis of Nonlinear Structures Using Sherman-Morrison Method

In this section it will be shown that if there is a localized non-linear element in a vibrational system, the type and parametric identification is possible if Sherman-Morrison matrix inversion formula is used in the expression for response amplitudes. Sherman-Morrison matrix inversion formula is first introduced by Sherman and Morrison [20] and it is applied to various vibrations and acoustics applications by Özer and Royston [21-24] and Hong and Kim [25].

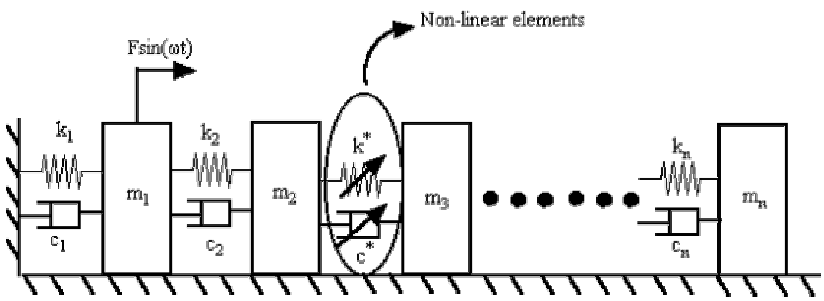


Figure 1. N-degree of freedom non-linear system with a localized non-linearity.

The system under consideration is shown in Figure 1. The non-linearity can be in between any of the degree of freedom of the system. Let us assume that the non-linearity exists between r^{th} and j^{th} coordinates. The non-linearity matrix $[\Delta]$ can be represented as:

$$[\boldsymbol{\Delta}] = \begin{bmatrix} \dots\dots\dots & 0 & \dots\dots\dots \\ : & : & : \\ : & : & : \\ 0 & \begin{bmatrix} \upsilon & -\upsilon \\ -\upsilon & \upsilon \end{bmatrix} & 0 \\ : & : & : \\ : & : & : \\ \dots\dots\dots & 0 & \dots\dots\dots \end{bmatrix} \tag{23}$$

Since non-linearity exists in between the rth and jth coordinates the number of non-zero elements of the [Δ] matrix is four. One can write the [Δ] matrix as a multiplication of two matrices as shown below:

$$[\boldsymbol{\Delta}] = \{\boldsymbol{\delta}_1\}\{\boldsymbol{\delta}_2\}^T \tag{24}$$

where

$$\{\boldsymbol{\delta}_1\} = \begin{bmatrix} 0 \\ : \\ \upsilon \\ -\upsilon \\ : \\ 0 \end{bmatrix} \quad , \quad \{\boldsymbol{\delta}_2\} = \begin{bmatrix} 0 \\ : \\ 1 \\ -1 \\ : \\ 0 \end{bmatrix} \tag{25)-(26}$$

It can be seen that only rth and jth elements of the above vectors are non-zero. Using the above equation in equation (8) which gives the non-linear response, one can obtain the following equation:

$$\{\mathbf{X}\} = \left[[\mathbf{Z}] + \{\boldsymbol{\delta}_1\}\{\boldsymbol{\delta}_2\}^{\mathrm{T}}\right]^{-1}\{\mathbf{F}\} \tag{27}$$

where definition of **[Z]** is given in equation (13).

Sherman-Morrison matrix inversion formula is given as

$$\left[[\mathbf{A}] + \{\mathbf{u}\}\{\mathbf{v}\}^{\mathrm{T}}\right]^{-1} = [\mathbf{A}]^{-1} - \frac{[\mathbf{A}]^{-1}\{\mathbf{u}\}\{\mathbf{v}\}^{\mathrm{T}}[\mathbf{A}]^{-1}}{1 + \{\mathbf{v}\}^{\mathrm{T}}[\mathbf{A}]^{-1}\{\mathbf{u}\}} \tag{28}$$

where **[A]** is N×N matrix with full rank and **{u}** and **{v}** are N×1 vectors. Then equation (27) can be written as

$$\{\mathbf{X}\} = \left([\boldsymbol{\alpha}] - \frac{[\boldsymbol{\alpha}]\{\boldsymbol{\delta}_1\}\{\boldsymbol{\delta}_2\}^{\mathrm{T}}[\boldsymbol{\alpha}]}{1 + \{\boldsymbol{\delta}_1\}^{\mathrm{T}}[\boldsymbol{\alpha}]\{\boldsymbol{\delta}_2\}}\right)\{\mathbf{F}\} \tag{29}$$

Recall that [α] is the receptance of the linear system and is given as:

$$[\boldsymbol{\alpha}] = [\mathbf{Z}]^{-1} \tag{30}$$

Using equation (29) one can write the response of each coordinate as follows:

$$\begin{bmatrix} X_1 \\ \vdots \\ X_r \\ X_j \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} X_{lin\,1} \\ \vdots \\ X_{lin\,r} \\ X_{lin\,j} \\ \vdots \\ X_{lin\,n} \end{bmatrix} - \frac{[\boldsymbol{\alpha}]\{\boldsymbol{\delta_1}\}\{\boldsymbol{\delta_2}\}^T}{1+\{\boldsymbol{\delta_1}\}^T[\boldsymbol{\alpha}]\{\boldsymbol{\delta_2}\}} \begin{bmatrix} X_{lin\,1} \\ \vdots \\ X_{lin\,r} \\ X_{lin\,j} \\ \vdots \\ X_{lin\,n} \end{bmatrix} \tag{31}$$

The non-linear responses are on the left hand-side of the equation and they are experimentally measured quantities. The vectors on the right hand-side of the equation are linear response of the system. They can be obtained as measured receptances of the system under low amplitude loading conditions where non-linearities would not be excited. The only unknown in this equation is ‘υ’ which is the describing function representation of the non-linearity. Equation (31) can be solved for the value of ‘υ’, and the values of ‘υ’ at different amplitudes of response can be plotted, from which the type and the parametric value of the non-linearity can be identified.

4. CASE STUDIES

4.1. Localization of Non-linearity in the Structure

It was explained in Section 3.1 that the describing function representation of the non-linearities and the non-linearity matrix concept can be used to identify the location of the non-linear elements. Unlike the type and parametric identification method developed for non-linearity, in this method there is no limitation in the number of non-linear elements; so the non-linearity can be distributed to many degrees of freedom. The localization procedure can be used to locate the “non-linearity infected” coordinates. As a case study, the system shown in Figure 2 is investigated. This is a mass-spring-damper system with three degrees of freedom. There is a non-linear macro-slip element (Coulomb friction) between the first and the second degree of freedoms. The numerical values of the system are given as follows:

M_1=1 kg, M_2=2 kg, M_3=1 kg, k_1=1000 N/m, k_2=1000 N/m, k_3=1000 N/m, k_4=1000N/m, c_1=8 Ns/m, c_2=2 Ns/m, c_3=2 Ns/m, c_4=2Ns/m, F=1 N, c_f=0.25 N (Coulomb friction force)

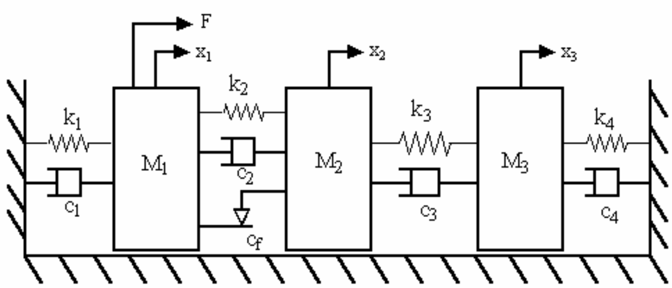


Figure 2. The three-degree of freedom system with a macro slip (Coulomb damping element)

In this case study, as well as in all others, the experimental results are simulated with the time domain numerical solution of the differential equation of motion. A fifth order Runge-Kutta numerical integration method is used to solve the equations of motion. The steady state response (amplitude and phase) information is obtained from the numerical solution. It is assumed that the steady state is reached after 40 cycles and the time step is determined by dividing each cycle into 400 equal step sizes (i.e. fixed step size is used). Linear responses are calculated by transforming the equations into frequency domain. Using equation (22) one can find the non-linearity numbers for each coordinate. The non-linearity numbers are plotted out for frequencies between 40 rad/sec and 80 rad/sec. Figures 3, 4 and 5 show the plots of the non-linearity numbers for each degree of freedom.

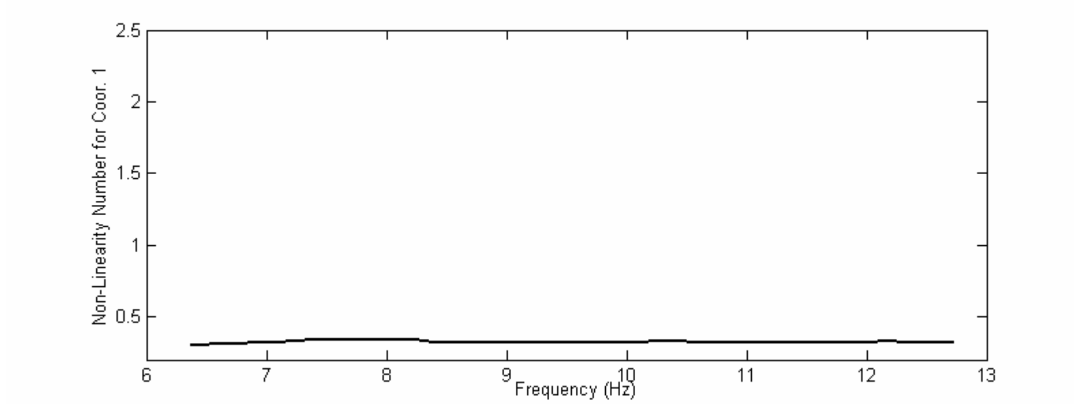


Figure 3. The value of the non-linearity number at coordinate 1 evaluated at different frequencies

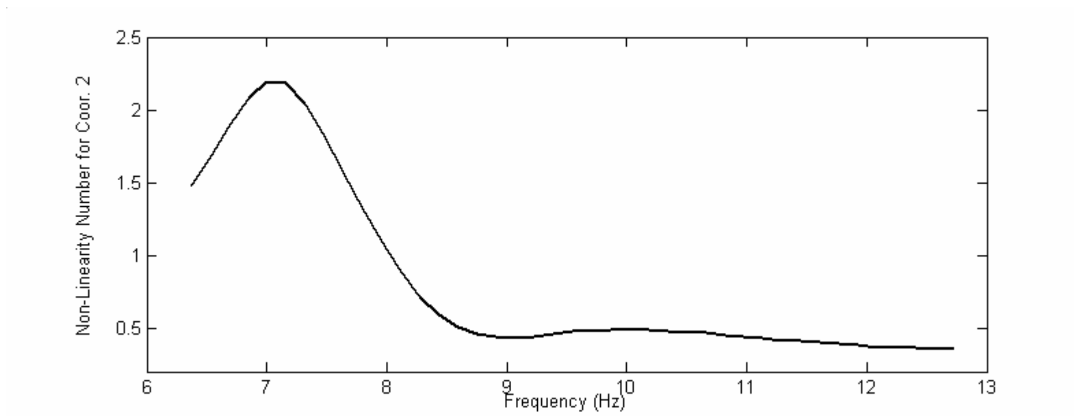


Figure 4. The value of the non-linearity number at coordinate 2 evaluated at different frequencies

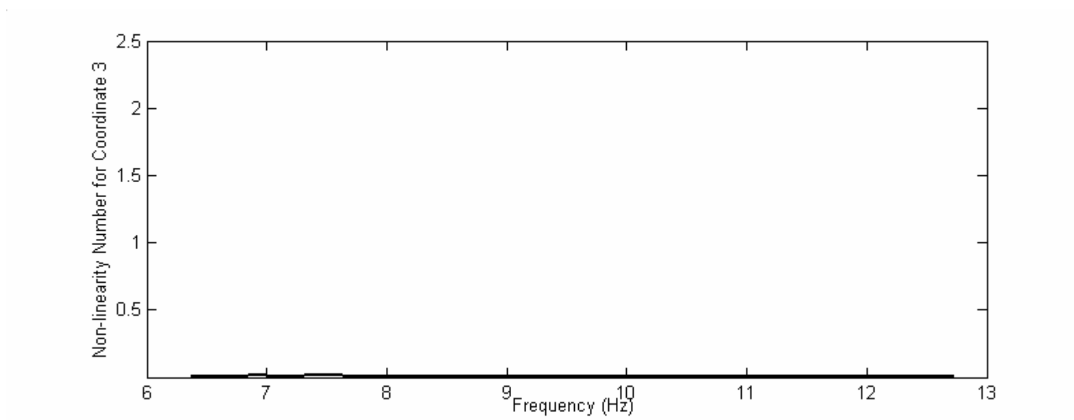


Figure 5. The value of the non-linearity number at coordinate 3 evaluated at different frequencies

From Figures 3 to 5 it can be observed that the non-linearity number has finite values at coordinates 1 and 2, while the value of the non-linearity number for coordinate 3 is very close to zero almost at all frequencies. The small values of the non-linearity number are due to not reaching to steady state during numerical solution. Figures 3 to 5 show that the coordinates to which a non-linear element is connected can be accurately identified using the method described in Section 3.1.

4.2. Type and Parametric Identification of Non-linearity: Single Degree of Freedom System

A single degree of freedom system with cubic type of stiffness non-linearity will be used in this case study to demonstrate that the type and parametric identification is possible applying the method suggested in Section 3.2. Experimentally measured response will be simulated with numerical solution of the differential equation of motion. The numerical data for the system in Figure 6 is given as below:

M=1 kg, k=100 N/m, $k^*=10^4$ N/m^3, c=1 Ns/m.

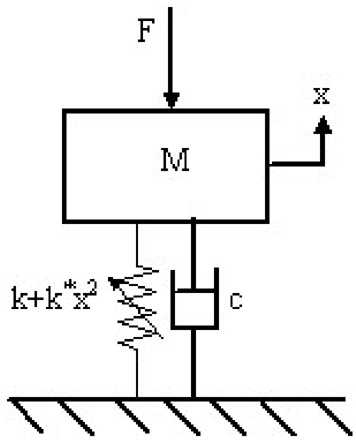


Figure 6. A single degree of freedom system with cubic stiffness

For a single degree of freedom system, using equation (29) one can find the Δ expression as follows:

$$\upsilon = \frac{\alpha - \theta}{\theta \alpha} \tag{32}$$

where θ is the 'pseudo receptance' of the non-linear SDOF system.

The receptance of the linear and non-linear system is given in Figure 7. Typical jump phenomenon can be observed in the non-linear response. The "non-linearity" value (Δ) (which is a scalar in this case) is calculated using the linear and non-linear receptance data. Delta values for different response amplitudes can be calculated using equation (32). The change of delta values with harmonic vibration amplitude is plotted in Figure 8. It can be seen from Figure 8 that the non-linearity is a polynomial type of non-linearity. The scalar value of delta is the describing function of the cubic type non-linearity. The describing function of a cubic type of stiffness non-linearity is given as:

$$\upsilon_{cubic} = \frac{3}{4} cs X^2 \tag{33}$$

where cs is the cubic stiffness coefficient and X is the response amplitude of the non-linear system. The solid line in Figure 8 is the data fit (obtained through non-linear regression) of the delta values calculated from equation (32). As depicted in Figure 8, the curve fit is closely tracing the numerically obtained delta values. Using equation (33) and the quadratic equation of the fit as given in Figure 9, one can identify the cubic stiffness coefficient to be 9764.5 N/m^3 which is very close to the cubic stiffness coefficient of the system: 10000 N/m^3. Figure 9 shows the response of the original non-linear system and the response of the identified system. It can be seen that the difference in the cubic stiffness coefficient result does not translate into an appreciable difference in the frequency domain response.

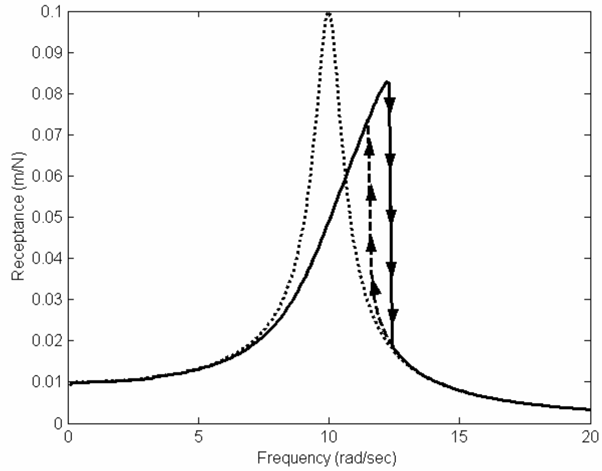


Figure 7. The receptance of the SDOF system case study, _ _ _ _ linear receptance, ____ non-linear “pseudo-receptance” (increasing frequency), __ __ __ __non-linear “pseudo-receptance” (decreasing frequency).

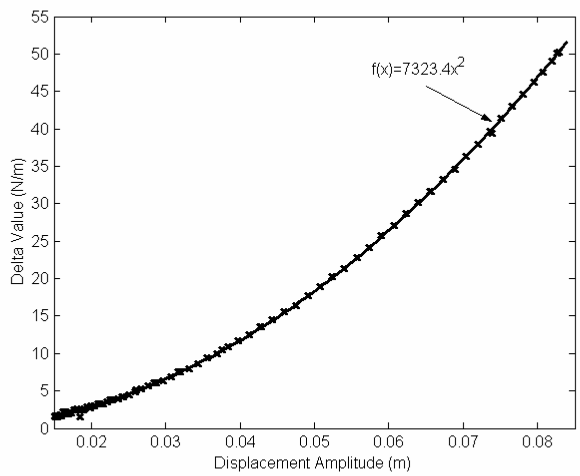


Figure 8. The delta values, ××××× calculated values, _____ obtained by performing a curve fit on calculated data.

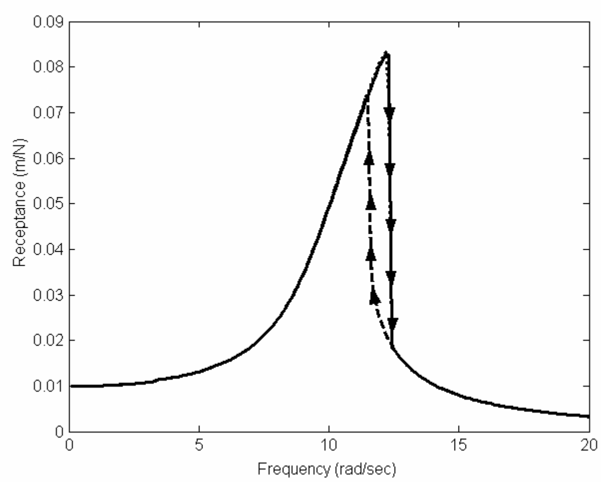


Figure 9. The receptance of, ____ original system, _ _ _ _ identified system.

4.3. Type and Parametric Identification of Non-linearity: Multi Degree of Freedom System

In this case study type and parametric identification of a multi degree of freedom system will be performed. The system that will be identified is the same system as the one in Section 4.1. Using equation (31) one can solve for the unknown describing function representation of the non-linearity.

Using equation (31) one can write the response of the k^{th} coordinate when the non-linearity is located between the r^{th} and j^{th} coordinates as follows:

$$X_k = X_{lin\,k} - \frac{(\alpha_{rr} - \alpha_{rj})\left(X_{lin\,r} - X_{lin\,j}\right)\upsilon}{1 + \upsilon\left(\alpha_{rr} - 2\alpha_{rj} + \alpha_{jj}\right)} \tag{34}$$

From equation (34) one can solve for the describing function representation of the non-linearity, υ, as shown below:

$$\upsilon = \frac{X_{lin\,k} - X_k}{\left(X_k - X_{lin_k}\right)\left(\alpha_{rr} - 2\alpha_{rj} + \alpha_{jj}\right) + \left(\alpha_{rr} - \alpha_{rj}\right)\left(X_{lin\,r} - X_{lin_j}\right)} \tag{35}$$

In this case study, the 'k' value (measured coordinate) is taken to be the first coordinate, the 'r' and 'j' values (the coordinates where the non-linear element is attached to) are 1 and 2, respectively.

Using the measured linear and non-linear responses of the k^{th} coordinate along with the linear receptance matrix, one can find the values of the describing function for that non-linearity type. Note that linear response can be obtained as the response of the non-linear system at a low forcing level. If the describing function of the non-linearity and the amplitude of the response are plotted, behavior of the curve allows us to identify the type of the non-linearity. If a simple curve fitting algorithm is applied the parametric identification is also possible.

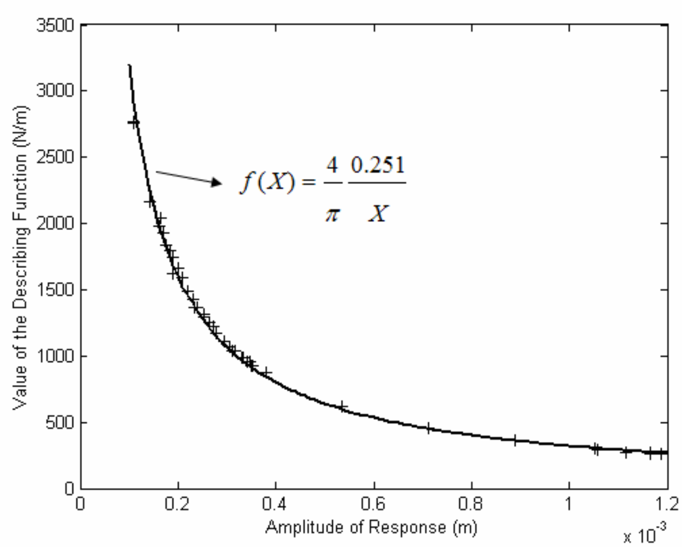


Figure 10. The plot of the calculated describing function values at different 'measured' response amplitudes.

It can be observed from Figure 10 that the describing function representation of the non-linearity decreases as the amplitude of the response increases. Table 1 shows that the only non-linearity that would cause such a behavior is the Coulomb damping type of non-linearity. Using the describing function representation for Coulomb damping one can fit a curve to data given in Figure 10. The curve fit (using non-linear regression analysis) gives a Coulomb friction force of 0.251 N which means less than %1 error in estimating the frictional force.

CONCLUSION

A new method is introduced for type and parametric identification of a non-linear element in an otherwise linear structure. This method is an extension of the method developed by the first two authors in an earlier work [10] to localize non-linearity in a multi degree of freedom system and to identify type and parameters of the non-linear element when it is located between the system and ground. The 'non-linearity matrix' concept is used also in this study, and the Sherman-Morrison matrix inversion method is applied to put the response expression in a form where the non-linearity term can be isolated. This made it possible to identify type and parameters of a non-linear element between any two coordinates of the system. Case studies are given to show that the method can be used to find the location of the non-linear element, and also to identify types of non-linearities successfully (cubic stiffness and Coulomb damping in the example cases given) along with parametric values of the non-linearities.

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